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LETTER TO THE EDITOR

Random and dendritic patterns in crack propagation

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Abstract. The transition from random fractal structures to dendritic patterns in crack propagation in an elastic medium is shown to be a consequence of averaging out fluctuations, as already found for the growth of structures described by scalar fields (such as for diffusion-limited aggregation or dielectric breakdown).

One of the most interesting topics in the physics of growing structures concerns the understanding of the conditions under which either random fractal structures (Mandelbrot 1983) or dendritic-like structures (Langer 1980) develop. In particular, the connection between tip-splitting phenomena and dendritic growth in diffusion-limited aggregation (DLA) and dielectric breakdown (DB), introduced by Witten and Sander (1983) and Niemeyer *et al* (1984) respectively, and viscous fingering, among others, has attracted a great deal of attention (Ben-Jacob *et al* 1985, Sander *et al* 1985, Kertész and Vicsek 1986, Nittmann and Stanley 1986, Meakin 1987a, b). It has been found that the asymmetry in fluctuations promoted by the growth process is responsible for the tip splitting which leads to the random fractal structures characteristics of DLA and DB. Reduction of noise by Monte Carlo averaging has facilitated the description of a continuous transition from tip splitting (dominated by fluctuations) to dendritic growth. The picture which emerges from these investigations (see Meakin (1987b) for a review) is that fluctuations reduce the effect of the anisotropy, leading to the formation of fractal-like patterns. When fluctuations are averaged out, dendritic-like patterns grow, with a regular anisotropic structure. Closely related to this interesting subject is the question recently raised by several authors (Meakin 1985, Ball and Brady 1985, Kertész and Vicsek 1986) concerning the anisotropy of aggregates grown on a lattice; according to these researchers, these aggregates are asymptotically not fractals because when their sizes are increased the anisotropy of the underlying lattice is gradually revealed.

Those investigations have all been carried out on structures growing in scalar fields. In this letter we shall study the transition from random fractals to dendritic structures in the case of crack propagation in elastic media. It has been shown recently (Louis and Guinea 1987a, Termonia and Meakin 1986) that fracture patterns in elastic media may have a fractal geometry, in close relationship with DLA and DB. In this work it will be shown that a transition to dendritic structures also occurs in this case as a consequence of averaging out fluctuations. In the following, we shall briefly outline the features of the model proposed by Louis and Guinea (1987a), pertinent to the present problem.

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We describe the elastic medium by a discrete lattice with nearest-neighbour central forces, namely

$$H = \tfrac{1}{2} \sum_{i,j} k_{ij}[(\boldsymbol{v}_i - \boldsymbol{v}_j) \cdot \hat{\boldsymbol{r}}_{ij}]^2 \tag{1}$$

where $\boldsymbol{v}_i$ is the displacement vector at site i, $\hat{\boldsymbol{r}}_{ij}$ is a unit vector between sites i and j and k_{ij} is the force constant. A triangular lattice is chosen in order to reproduce the correct isotropic properties in the continuum limit. In this case the equilibrium equations are

$$(\lambda + \mu)\partial_i \left(\sum_j \partial_j v_j\right) + \mu \left(\sum_j \partial_j^2\right) v_i = 0 \tag{2}$$

where λ and μ are the two Lamé coefficients, $\boldsymbol{v}(\boldsymbol{r})$ is the displacement field and ∂_i is the partial derivative with respect to the ith component of $\boldsymbol{r}$. In the present work all force constants between lattice points are taken equal to unity; this case corresponds to a ratio between Lamé coefficients $\lambda/\mu = 1$ (Louis and Guinea 1987a).

Equation (2) is a generalisation of the Laplace equation which describes processes like DLA (Witten and Sander 1983), DB (Niemeyer *et al* 1984) and viscous fingering in 2D cells (Nittmann *et al* 1985). The close relationship between those three non-equilibrium phenomena and the present one has been discussed previously by Louis and Guinea (1987a); the main difference consists in the vectorial nature of the field characteristic of the present problem. It can therefore be expected that, as already found for structures growing in scalar fields on a lattice, the anisotropy of the lattice be revealed for very large structures. In the present work we are concerned with rather small structures and therefore we only investigate the connection between anisotropy and fluctuations.

In order to allow a crack to grow we must apply forces at the external boundaries of the medium to induce a finite distribution of stresses within it. We will consider the case of uniform dilation. We initiate the crack by making the force constant of a given bond on the lattice equal to zero. Then the nodes of the lattice are displaced to achieve equilibrium; the displacements of the nodes which lie at a rhombus far enough from the crack are kept fixed. The stresses accumulated in the bonds adjacent to the broken bond are then calculated. The ten next-nearest neighbours of the broken bond are considered as candidates for breaking. Other choices are possible such as four or six neighbours; nonetheless, although this may affect the characteristics of the pattern in the initial stages of the process (Meakin 1987c, Louis and Guinea 1987b), it is not yet clear whether it has any influence on rather large cracks containing many broken bonds.

Noise reduction was accounted for, as previously done for structures growing in scalar fields (Meakin 1987a). From the set of candidates, a particular bond was picked with probability proportional to the absolute value of the stress, but only when it has been chosen a given number of times (s) is its force constant set equal to zero; s is a parameter which can be tuned. In order to differentiate more clearly between structures growing with different values of s from the very early stages of growth, we start by breaking all bonds contained in a hexagon of side 5; this procedure has no influence on large structures.

Figures 1–3 illustrate the kind of structures obtained for several values of the averaging parameter s, ranging from 1 to 1000. The early stages of growth are shown in figure 1. We note a very remarkable difference between the structures. For $s = 1$ (figure 1(a)) we recover random fractal growth as already discussed by Louis and

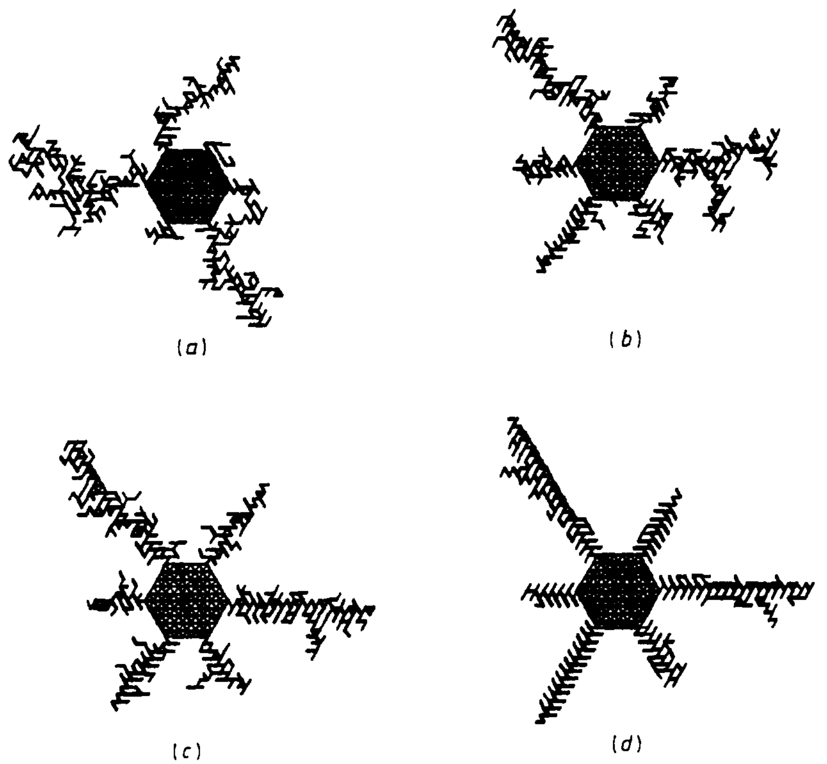

Figure 1. Cracks containing around 550 broken bonds, propagating in an elastic medium with $\lambda = \mu$. The crack was started by breaking all bonds in a hexagon of side 5. Structures grown with different degrees of averaging are shown: (*a*) $s = 1$, random fractal, (*b*) $s = 5$; (*c*) $s = 10$; (*d*) $s = 80$.

Guinea (1987a). When s is increased tip splitting is reduced (figure 1(*b*)) and nearly eliminated for $s > 10$ (figure 1(*c*)). For a large value of the noise parameter, $s = 80$, the dendritic nature of the pattern is clearly visible (figure 1(*d*)).

Cracks containing over 1000 broken bonds are shown in figures 2 and 3. First we note that $s = 5$ represents too low a reduction of fluctuations to eliminate tip splitting (figure 2(*a*)) as already noted in smaller patterns. Here we should remark that the asymmetry of the patterns in figures 2 and 3 arises from the amplification of small initial fluctuations which cannot be avoided even in the large-s limit. Patterns showing a remarkable dendritic-like structure are already obtained for $s = 20$ (figure 2(*b*)). The crack shown in that figure has a distinguishable dendritic nature; tip splitting is very scarce.

Finally we address a question recently raised by Kertész and Vicsek (1986). According to these authors, for very large averaging a second transition from dendritic structures to needle crystals takes place in DLA; they reported needle-like patterns for s around 400. In figure 3 we show cracks containing around 1600 broken bonds for $s = 80$ and 1000. It is remarkable that both structures are very similar and no indication of the transition to needle crystals is observed. Although we do not have any sound explanation for the discrepancy between our results and those of Kertész and Vicsek, it is worth remarking on the vectorial nature of the fields in which our structures have been grown, in contrast with the scalar character of the field in DLA or DB.

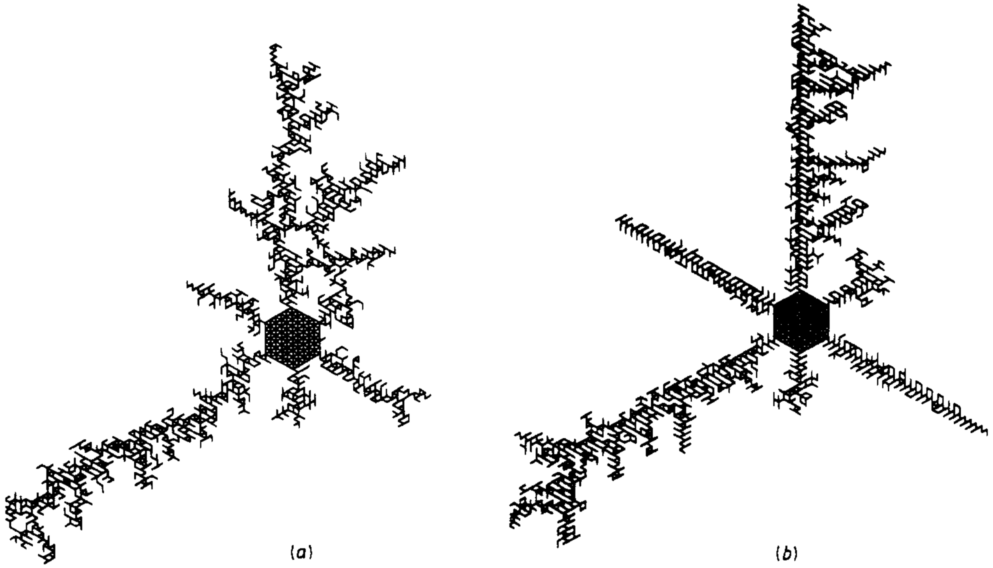

Figure 2. Same as figure 1 for (a) $s = 5$ and around 1900 broken bonds; (b) $s = 20$ and around 1900 broken bonds.

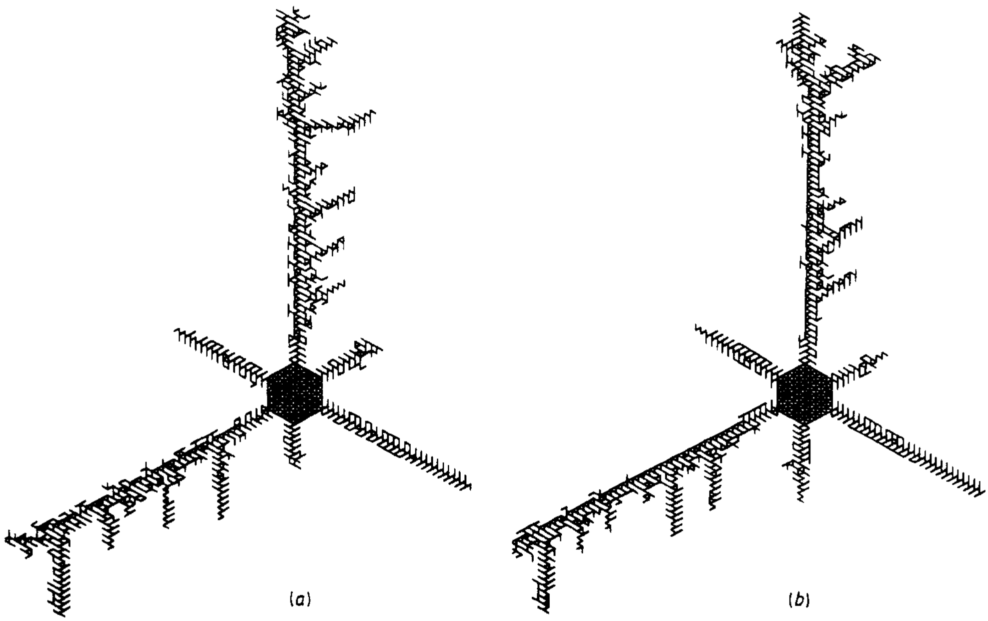

Figure 3. Same as figure 1 for (a) $s = 80$ and 1650 broken bonds; (b) $s = 1000$ and 1600 broken bonds.

In this letter we have investigated the transition from random fractal structures to dendritic patterns in crack propagation. As previously found for DLA or DB, this was again found to be a consequence of fluctuations; for large fluctuations fractal structures grow but when averaging is introduced cracks tend to have a dendritic-like character instead. For very strong averaging no transition to needle crystals was found.

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References

Ball R C and Brady R M 1985 *J. Phys. A: Math. Gen.* **18** L809
Ben-Jacob E, Grodbey R, Goldenfeld N D, Koplik J, Levine H, Mueller T and Sander L M 1985 *Phys. Rev. Lett.* **55** 1315
Kertész J and Vicsek T 1986 *J. Phys. A: Math. Gen.* **19** L257
Langer J S 1980 *Rev. Mod. Phys.* **52** 1
Louis E and Guinea F 1987a *Europhys. Lett.* **3** 871
—— 1987b unpublished
Mandelbrot B B 1983 *The Fractal Geometry of Nature* (San Francisco: Freeman)
Meakin P 1985 *J. Phys. A: Math. Gen.* **18** L661
—— 1987a *Preprint*
—— 1987b *Preprint*
—— 1987c private communication
Niemeyer L, Pietronero L and Wiesmann H J 1984 *Phys. Rev. Lett.* **52** 1033
Nittmann J, Daccord G and Stanley H E 1985 *Nature* **314** 141
Nittmann J and Stanley H E 1986 *Nature* **321** 663
Sander L M, Ramanlal P and Ben-Jacob E 1985 *Phys. Rev.* A **32** 3160
Termonia Y and Meakin P 1986 *Nature* **230** 429
Witten T A and Sander L M 1983 *Phys. Rev.* B **27** 5686